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Reflexive IW Model II

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14. ABSTRACT

The results of advancing the use of reflexive processes in a multiple agent environment are presented. Emphasis is placed on a mathematical representation.

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1. INTRODUCTION

The objective of this research is to expand a Reflexive IW Model $<x_1,x_2,x_3>$ to a model $<x_1,x_2,M>$, where M is the agent's model of a situation. (A description of the $<x_1,x_2,x_3>$-model is given in the following works: Lefebvre (1982, 1992, 1999a,b), Krylov (1994), Schreider (1994), Adams-Weber (1997), Miller & Sulkoski (1999a,b), Taran (1999)). The $<x_1,x_2,M>$-model allows the agent to operate with *higher* values, which do not have intensity and can be characterized only by binary evaluations "positive" or "negative." We show in detail how to construct a functional representation of M using objective information. The values of x_1 and x_2 reflect "lower" values, utility-measures of the alternatives which have intensities and can be associated with real numbers. One of the central problems in reflexive modeling is to elaborate methods for finding the values of x_1 and x_2. We show in this work that we can expand an $<x_1,x_2,M>$ model in such a way that finding the values of x_1 and x_2 can be reduced to solving standard problems from the classical game theory.

In the process of this research, it was *unexpectedly discovered* that, for modeling strategical decisions when all the variables in the model take on only values of 1 and 0, *there is, in principle, no need* to have preliminary information about the values of x_1 and x_2. This is due to the fact that finding strategical decision can be reduced to solving *functional* equations, not numerical ones.

In this work we analyzed in detail the possibility of using the model $<x_1,x_2,M>$ in conducting *reflexive control*, i.e., during informational influence on an enemy with the purpose of inclining him to make a decision favorable for the party conducting such control.

All illustrative examples are chosen in such a way that the positive pole is bound up with an active stance on the part of the agent, and the negative pole with a passive stance. Consistently with the notation introduced in our previous report (Lefebvre, 1999b), the actual pressure toward the positive pole is designated as p_1 and the expected pressures as p_2. Thus, $p_1=x_1$ and $p_2=x_2$. It is necessary to emphasize that the $<x_1,x_2,M>$-model is *intentional*. This means that the agent's subjective *will* is considered as an objective factor, represented in the model by a special *variable*. This is an essential difference between the reflexive models and all other methods of modeling the decision-making process. The sophistication of modern reflexive modeling allows us already today to use this method in the real analysis of possible enemy actions as well as in preparing an informational attack.

2. REFLEXIVE MODELS

In this report, we introduce an $<x_1,x_2,M>$-model of an agent whose objective is to choose between two alternatives. One of these alternatives embodies *good* for the agent, and the other embodies *bad*. We will call these alternatives the *positive* and *negative* poles. Our implicit supposition that the agent is capable of desiring or planing to act. We call this ability will or *intention*. It is not always the case, however, that an intention turns into a real action; an example is when a person who has decided to rob a bank does not do so because he cannot bring himself to aim his pistol at the teller. Thus, we introduce an additional characteristic: the agent's *readiness* to accomplish an action. **Intention** is a phenomenon of the agent's mental domain, whereas **readiness** is the attribute of an executive system in the agent, not fully under the control of his will.

Within the framework of this model, intention and readiness have *measures*. This means that they correspond to the elements of a set E on which at least a *relation of reflexivity* is defined, i.e., each element is identical to itself. If we designate the measures of intention and readiness as $|\,intention\,|$ and $|\,readiness\,|$, then the model being developed in this report can be represented as the following function:

$$|\,readiness\,| = F_D(|\,intention\,|), \tag{2.1}$$

where parameter D embodies a set of factors influencing the agent's choice.

If we designate $|\,intention\,|$ as x and $|\,readiness\,|$ as y, then equation (2.1) appears as $y=F_D(x)$. Consider an example. An agent is facing the alternatives of helping a friend or of withholding help. The agent's intention can be expressed in these words:

I plan to help my friend.

The agent's readiness, on the other hand, is expressed in the words of an external observer:

He will help his friend.

In the given case the set E is $E=\{0,1\}$, where 0 and 1 are the elements of Boolean algebra: 0 corresponds to the absence of help, and 1 to its actualization. In this example the measure of intention is Boolean 1, ($x=1$), and the measure of readiness is Boolean 1, ($y=1$), but in other cases they may be different. If from an external observer's point of view,

He will not help his friend,

then the measure of readiness is 0, ($y=0$), while the measure of intention remains 1.

In our model, intention is characterized only by the measure described above; we assume that the intentions with equal measures cannot be distinguished by the agent's cognitive system which elaborates decisions. The same can be said about readiness. This allows us to consider set E as a set of alternatives. From now on we will omit the word "measure," and use only the terms *intention* and *readiness*.

We call a choice *intentional*, if the value of $x=x^*$ is such that the following equation holds

$$x^*=F_D(x^*), \tag{2.2}$$

that is, x^* is a fixed point of transformation F_D.

Under condition (2.2) the intention "turns into" readiness, that is, the agent is ready to choose the action he desires to accomplish. When $x \neq y$, the agent's desire does not coincide with his readiness to fulfill this particular choice. If F_D has no fixed points, this means that the agent cannot make an intentional choice under given circumstances. Equation (2.2) allows us to include the concept of *free choice* in our model. The necessary condition for free choice is the existence of at least two different fixed points in the transformation $F_D(x)$. A formal analogue to free choice is the selection of one of the fixed points by the agent under conditions that the agent has no criterion for preferring one of them over the other (Lefebvre, 1992)

The model also introduces a formal analogue to *free will*. This appears in the assumption that the variable x can take on any value from E -- in other words, that the agent can have any desire from the set E regardless of its potential to be turned into the readiness. Thus, the concepts of free choice and free will are distinguished.

Within the framework of reflexive models, the transformation $F_D(x)$ does not necessarily have fixed points; that is, there may be states in which the agent cannot make an intentional choice.

A key problem connected with all reflexive models lies in indicating a specific form of the function $y=F_D(x)$. Using simple but fundamental assumptions concerning human choice, we demonstrated (Lefebvre, 1999a) that the agent corresponds to the following function:

$$y=x_1+(1-x_1)(1-x_2)c(x), \tag{2.3}$$

where x, x_1, x_2, and $c(x)$ take on the values from the interval [0,1]. The model $<x_1,x_2,x_3>$, which was introduced earlier, is a special case of the $<x_1,x_2,M>$ model where $c(x)\equiv x$. If the domain of the variables x_1, x_2, x, and $c(x)$ consists not of all points from the interval [0,1], but only of its extremes 1 and 0, then function (2.3) can be replaced with the following Boolean function:

$$A_1 = a_1 + \bar{a}_2 C(x_3)\,. \tag{2.4}$$

The variables in (2.4) and (2.3) correlate as follows:

$$a_1 \leftrightarrow x_1$$
$$a_2 \leftrightarrow x_2$$
$$x_3 \leftrightarrow x$$
$$C(x_3) \leftrightarrow c(x).$$

Variable a_1 represents the *actual pressure* toward the positive pole at the moment of choice, a_2 represents the *expected pressure*, x_3 is the agent's *intention* or *plan*, and function $C(x_3)$ shows the agent's *cognizant evaluation of the situation*. We presume that $C(x_3)$ has a form of $W(x_3,B_3)$, where B_3 is the first agent's *cognizant image of the second agent*.

We will demonstrate further that a Boolean model can be used for modeling, elaborating, and choosing the *plan* of the consequent actions; we will call this kind of the agent's activity -- the *metachoice*. The general model $<x_1,x_2,M>$ can be used for modeling an actual bipolar choice.

3. METACHOICE

The essence of a metachoice, corresponding to transformation $|readiness|=F_D(|intention|)$, consists in generation and choice of a *program* for bipolar choice which will be carried out in the future. In metachoice, $|readiness|$ and $|intention|$ take their values not from Boolean set {0,1}, but from the set Φ of Boolean functions $\Phi(a,b)$, each of which corresponds to a certain program. An agent cannot obtain information about his real readiness directly; it is represented in his inner domain as intention. Thus, only under the condition that the same Boolean function x^* corresponds to both intention and readiness does the agent have a correct view of the program it is prepared to implement. In other words, valid information about the program of bipolar choice appears in the agent only through his actual intentional choice of a program; this corresponds to $x^*=F_D(x^*)$.

During the process of metachoice the agent corresponds to the following equation:

$$A_1(a_1,a_2,B_3) = a_1 + \bar{a}_2 W(x_3 B_3)\,, \tag{3.1}$$

where

$$x_3=x_3(a_1,a_2,B_3). \tag{3.2}$$

Therefore, $|readiness|=A_1(a_1,a_2,B_3)$, $|intention|=x_3(a_1,a_2,B_3)$, $F_D = a_1 + \bar{a}_2 W(x_3,B_3)$, D={Boolean function $W(a,b)$, Boolean variable a_1, Boolean variable a_2, Boolean variable B_3}.

Let us consider an *intentional* metachoice. The agent's state $\psi(a_1, a_2, B_3)$ corresponds to the following equation:

$$a_1 + \bar{a}_2 W(\psi(a_1, a_2, B_3), B_3) \equiv \psi(a_1, a_2, B_3), \tag{3.3}$$

which follows from (3.1) under the condition that

$$A_1 = x_3 = \psi(a_1, a_2, B_3), \tag{3.4}$$

where ψ is an unknown function.

An intentional choice is possible only if (3.3) has at least one solution ψ.

Let us investigate now the conditions under which equation (3.3) has solutions. The role of $W(x_3, B_3)$ can be played by any of the 16 Boolean functions of the type $\Phi(a,b)$, where $a=x_3$ and $b=B_3$, given in the Table 3.1.

We will consider that $\alpha \geq \beta$, if the case of $\alpha=0$ and $\beta=1$ is excluded. Function $\Phi(a,b)$, will be called non-decreasing along a, or *ND*-function, if with $\alpha \geq \beta$, $\Phi(\alpha,b) \geq \Phi(\beta,b)$, where $b \in \{0,1\}$. In the opposite case, function $\Phi(a,b)$ will be called a *D*-function. The analysis of all 16 Boolean functions $W(x_3, B_3)$ leads to the following list of *ND*- and *D*-functions (the numbers in Table 3.2 coincide with the numbers given these functions in Table 3.1).

Table 3.1
Sixteen Boolean functions of the type $\Phi(a,b)$

1	0	9	$\bar{a}\bar{b}$
2	a	10	$a+b$
3	$\bar{a}$	11	$a+\bar{b}$
4	b	12	$\bar{a}+b$
5	$\bar{b}$	13	$\bar{a}+\bar{b}$
6	ab	14	$\bar{a}b+a\bar{b}$
7	$\bar{a}b$	15	$ab+\bar{a}\bar{b}$
8	$a\bar{b}$	16	1

Table 3.2
ND- and *D*-functions

ND-functions		*D*-functions	
1	0	3	$\bar{x}_3$
2	x_3	7	$\bar{x}_3 B_3$
4	B_3	9	$\bar{x}_3 \bar{B}_3$
5	$\bar{B}_3$	12	$\bar{x}_3 + B_3$
6	$x_3 B_3$	13	$\bar{x}_3 + \bar{B}_3$
8	$x_3 \bar{B}_3$	14	$\bar{x}_3 B_3 + x_3 \bar{B}_3$
10	$x_3 + B_3$	15	$x_3 B_3 + \bar{x}_3 \bar{B}_3$
11	$x_3 + \bar{B}_3$		
16	1		

Statement 3.1. Equation

$$a_1 + \bar{a}_2 W(x_3, B_3) \equiv x_3 \tag{3.5}$$

has a solution if and only if $W(x_3,B_3)$ is a *ND*-function.

The ***proof*** consists in direct analysis of all the 16 Boolean functions. It follows from Statement 3.1 that if $W(x_3,B_3)$ is a *D*-function, then equation (3.5) has no solution. Table 3.3 gives solutions for all equations in which $W(x_3,B_3)$ is an *ND*-function.

In the preceding analysis, variables a_1 and a_2 were treated as independent. There can, however, be various functional relations between them, reflecting connections between past and present events. For example, a_1=a_2. This case represents a situation containing no surprises. The world today exercises the same pressure on the agent as it did yesterday.

Table 3.3
Solutions of Equations $a_1 + a_2 W_i(x_3, B_3) \equiv x_3$,
where $W_i(x_3,B_3)$ is an *ND*-function

i	*ND*-function W_i	Set of solutions $x_3=\psi(a_1,a_2,B_3)$	Solutions independent of B_3
1	0	$\psi_1 = a_1$	$\psi_1 = a_1$
2	x_3	$\psi_1 = a_1 + \bar{a}_2, \psi_2 = a_1,$ $\psi_3 = a_1 + \bar{a}_2 B_3, \psi_4 = a_1 + \bar{a}_2 \bar{B}_3$	$\psi_1 = a_1 + \bar{a}_2, \psi_2 = a_1,$
4	B_3	$\psi_1 = a_1 + \bar{a}_2 B_3$	
5	$\bar{B}_3$	$\psi_1 = a_1 + \bar{a}_2 \bar{B}_3$	
6	$x_3 B_3$	$\psi_1 = a_1, \psi_2 = a_1 + \bar{a}_2 B_3$	$\psi_1 = a_1$
8	$x_3 \bar{B}_3$	$\psi_1 = a_1, \psi_2 = a_1 + \bar{a}_2 \bar{B}_3$	$\psi_1 = a_1$
10	$x_3 + B_3$	$\psi_1 = a_1 + \bar{a}_2, \psi_2 = a_1 + \bar{a}_2 B_3$	$\psi_1 = a_1 + \bar{a}_2$
11	$x_3 + \bar{B}_3$	$\psi_1 = a_1 + \bar{a}_2, \psi_2 = a_1 + \bar{a}_2 \bar{B}_3$	$\psi_1 = a_1 + \bar{a}_2$
16	1	$\psi_1 = a_1 + \bar{a}_2$	$\psi_1 = a_1 + \bar{a}_2$

4. BASIC EXAMPLES OF METACHOICE

An international criminal syndicate has committed brutal acts against the citizens of countries **A** and **B**. Each country may demand from the UN discuss the situation and indict the syndicate for crimes against humanity, or it may remain silent. The UN session will take place in two months. The governments of **A** and **B** believe that indictment is a positive outcome and that silence is a negative outcome. The UN session will issue a declaration only if both countries **A** and **B** demand this. The criminal syndicate may, however, threaten governments **A** and **B**. Such a threat can be made to country **A** only during secret meetings between representatives of the country and of the criminal syndicate. There can be only two such meetings: first, a month before the UN session, the other an hour prior to the session. The syndicate's goal is to block the UN's declaration. **A**'s government would like to make strategic decision in advance, taking into consideration all feasible versions of future events, as they are seen by **A** two month before the UN session.

Let us model the generation of a set of decisions by country **A**. From the government's point of view, the sought-for UN indictment is a positive action, whereas silence is negative, so that $x_3=1$ can stand for the intention to demand the indictment, and $x_3=0$ is the intention to remain silent. $B_3=1$ is country **B**'s decision (from country **A**'s point of view) to speak up on indictment, and $B_3=0$ is **B**'s decision to remain silent. Function $W(x_3,B_3)$ is **A**'s model of the UN. If $W=1$, the UN declares its indictment, if $W=0$, it does not. The values of a_2 and a_1 correspond to the syndicate's behavior during the first and second meetings respectively: $a_2=1$ means there was no threat during the first meeting, and $a_2=0$ means there was a threat; $a_1=1$ means there was no threat during the second meeting, and $a_1=0$ means there was a threat. Since the UN will issue a declaration only if both countries demand this, a table for function W is as follows:

Table 4.1

x_3	B_3	$W(x_3,B_3)$
1	1	1
1	0	0
0	1	0
0	0	0

It follows from this table that

$$W(x_3,B_3)=x_3B_3, \tag{4.1}$$

and equation (3.1) acquires a form of

$$a_1 + \bar{a}_2 x_3 B_3 = A_1 . \tag{4.2}$$

For the intentional choice, equation (4.2) turns into

$$a_1 + \bar{a}_2 x_3 B_3 \equiv x_3 , \tag{4.3}$$

where $x_3 = \psi(a_1, a_2, B_3)$.

Equation (4.3) has a solution, since function (4.1) is an *ND*-function (see Table 3.2). Let us demonstrate a process of solving this equation. First, note that when a_1 and a_2 are not equal to 0 simultaneously, the value of x_3 is either 1 or 0. When,

$$a_1 = a_2 = 0, x_3 = \psi(0,0,B_3) .$$

We now construct a table connecting the values of $\psi(a_1, a_2, B_3)$ with those of a_1 and a_2. For the values of function ψ not equal to 0, the conjunctive members of its disjunctive normal form are given below:

Table 4.2

a_1	a_2	x_3	conjuction
1	1	1	$a_1 a_2$
1	0	1	$a_1 \bar{a}_2$
0	1	0	
0	0	$\psi(0,0,B_3)$	$\bar{a}_1 \bar{a}_2$

Connecting these conjunctions with + and making simplifications, we find that

$$\psi = a_1 + \bar{a}_2 \psi(0,0,B_3) B_3 . \tag{4.4}$$

Function $\psi(0,0,B_3)$ depends on one Boolean variable B_3 and may be one of the following four functions: 1, 0, B_3, $\bar{B}_3$. By substituting these values into (4.4) we find that equation (4.3) has two different roots:

$$(1)\ \psi_1 = a_1.$$

$$(2)\ \psi_2 = a_1 + \bar{a}_2 B_3.$$

Interpretation of ψ_1:

If during the second meeting (an hour before the UN session), there is no threat, (a_1=1), we will demand indictment, $(\psi_1 = 1)$, regardless of what happens at the first meeting. If we receive a threat during the second meeting, (a_1=0), we will remain silent, $\psi_1 = 0$.

Interpretation of ψ_2:

If during the second meeting we do not receive a threat, (a_1=1), we will demand indictment, $(\psi_1 = 1)$. If we receive a threat, (a_1=0), our reaction will depend on circumstances. A surprise threat (i.e.not preceded by a threat during the first meeting, a_2=1) will entail our silence. If we are prepared to a threat (i.e. there was a threat during the first meeting, a_2=0), then our reaction will depend on **B**'s behavior: if **B** demands the indictment, (B_3=1), we will also do so, $(\psi_2 = 1)$, if **B** remains silent, (B_3=0), so do we, $(\psi_2 = 0)$.

It is worth noting that we do not consider the procedure of solving equation (3.3) as a logical deduction performed by the agent. We assume that this procedure models an automatic process in the agent's cognitive system generating the set of programs for bipolar choice.

Let us consider now another situation. Even though both countries, **A** and **B**, suffered from the syndicate's brutal actions, only **A**'s demand will produce an indictment by the UN. In this case, $W(x_3,B_3) \equiv x_3$, and an equation representing the intentional choice is

$$a_1 + \bar{a}_2 x_3 \equiv x_3. \tag{4.5}$$

Function $W(x_3,B_3) \equiv x_3$ is a *ND*-function, thus, equation (4.5) has a solution. From Table 3.3 we find four solutions of the type $\psi = \psi(a_1, a_2, B_3)$ for this equation:

$$(1)\ \psi_1 = a_1 + \bar{a}_2.$$

$$(2)\ \psi_2 = a_1.$$

$$(3)\ \psi_3 = a_1 + \bar{a}_2 B_3.$$

$$(4)\ \psi_4 = a_1 + \bar{a}_2 \bar{B}_3.$$

Let us interpret these solutions.

Interpretation of $\psi_1 = a_1 + \overline{a}_2$:

If there are no threats immediately before the UN session, (a_1=1), we will demand indictment, $(\psi_1 = 1)$; if we receive a threat, (a_1=0), our actions will depend on the circumstances. If this threat comes as a surprise, (a_2=1, i.e., no threats were received in the past), we will be silent $(\psi_1 = 0)$; if we are prepared for it, (a_2=0, i.e., the threat was received during the first meeting between our representatives and those of the syndicate), we'll demand indictment, $(\psi_1 = 1)$.

Interpretation of $\psi_2 = a_1$ (identical to the interpretation of ψ_1 in the previous example):

Our decision will depend on the events preceding the UN session. If there are no threats at this moment, (a_1=1), we will demand the indictment, $(\psi_2 = 1)$; if we receive a threat, (a_1=0), we will remain silent, $(\psi_2 = 0)$.

The program $\psi_3 = a_1 + \overline{a}_2 B_3$ has **the same interpretation** as program ψ_1 in the previous example.

Interpretation of $\psi_4 = a_1 + \overline{a}_2 \overline{B}_3$:

It almost completely recapitulates program ψ_3, but when a_1=0, a_2=0 the agent thinks:

I'll make a decision opposite to the one made by **B**, from my point of view, $\psi = \overline{B}_3$.

This raises an interesting point. In the last example, the UN decision to condemn the syndicate does not depend on **B**'s bipolar choice (function $W(x_3,B_3)\equiv x_3$ does not depend effectively on B_3). Yet the very fact that **B** is present in **A**'s model of the situation (B_3 is one of the arguments of $x_3 = \psi(a_1, a_2, B_3)$)leads to the dependence of **A**'s actions on **B**'s actions in the two programs (B_3 is an effective argument of the functions ψ_3 and ψ_4).

After the agent's cognitive system has completed its generation of the set of programs $\{\psi_1, \psi_2, \psi_3, \psi_4\}$, the agent realizes his ability to free choice and selects one of them. The choice of a program can be performed long before the UN session, so that during the session **A** merely carries out a decision made earlier.

Let us emphasize that the model allows us to find a set of possible programs for **A**'s future actions; it does not allow us, however, to determine which specific program **A** will choose. The freedom of choice cannot be modeled within the framework of reflexive models.

5. THE APPEARANCE OF A DILEMMA

There are **four types** of relations between the values of W and those of x_3 and B_3. **First**, function W may be independent of both x_3 and B_3, i.e. it can be always equal to either 0 or 1. In this case the future is not causally related to the agents' present actions. **Second**, W may be independent only of x_3, i.e., the value of W depends only of B_3. In this case the future does not depend on **A**'s actions in the present. **Third**, it is possible that W is independent of B_3. This means that the future is not related causally to **B**'s present actions. **Finally**, W depends on both x_3 and B_3; this means that there is a causal relation between both agents' present actions and a version of the future yet to be realized. In the last case there might be situations in which one agent's choice of actions determines the future and the other agent's actions are inessential. For example, if $W=x_3B_3$, then for $B_3=0$, $W=0$ independently of the values taken on by x_3.

Let the following identity hold for a fixed value $B_2 = \beta$:

$$W(x_3, \beta) \equiv \overline{x}_3, \tag{5.1}$$

where $x_3 \in \{0,1\}$. Suppose that the agent's *goal* is to arrive at a situation which can be evaluated positively. His *means* for achieving this is to perform an action (in this case a mental one) corresponding to the value of x_3. It follows from (5.1) that the situation will be positive only if the agent performs a negative action, because for $W=1$, $x_3=0$. Thus, equation (5.1) models a situation in which, for achieving a *positive* goal, one must employ a *negative* means.

We will call function $W(x_3,B_3)$ *anti-intentional*, if identity (5.1) holds for a certain value β. The analysis of all 16 functions $W(x_3,B_3)$ can be summarized as follows:

Statement 5.1. Function $W(x_3,B_3)$ is anti-intentional if and only if it is a D-function.

It was proven earlier (see Statement 3.1) that if $W(x_3,B_3)$ is a D-function, equation $a_1 + \overline{a}_2 W(x_3, B_3) \equiv x_3$ has no solutions. It follows from Statement 5.1 that this equation does not have solutions precisely in those cases where condition (5.1) holds for function $W(x_3,B_3)$; when, in other words, the situation described by function $W(x_3,B_3)$ contains a case in which a positive goal can be reached only by using negative means. We will call the condition (5.1) a "moral dilemma." Now we can say that the agent finds himself in a deadlock, i.e., he cannot effect metachoice in a situation containing a moral dilemma.

We will change now our basic example. Let $W(x_3,B_3)$ be a model of the world community. Let world public opinion become indignant at the syndicate's actions, if country **A** remains silent during the UN session, because it would mean that **A** is intimidated by the criminals. On the other hand, if country **A** demands indictment, there will be no significant reaction by the world community. In this case, function W is given by the following table:

Table 5.1

x_3	B_3	$W(x_3,B_3)$
1	1	0
1	0	0
0	1	1
0	0	1

It follows from this table that

$$W(x_3, B_3) = \bar{x}_3\,. \tag{5.2}$$

Then (3.1) has the form

$$a_1 + \bar{a}_2 \bar{x}_3 = A_1, \tag{5.3}$$

and, for metachoice the equation is

$$a_1 + \bar{a}_2 \bar{x}_3 \equiv x_3\,. \tag{5.4}$$

Function $W(x_3, B_3) \equiv \bar{x}_3$ is a *D*-function, thus (5.4) has no solution. *In such a situation A cannot formulate a set of programs for choice in advance.*

The model of the agent allows us to propose a new approach to the well-known prisoner's dilemma, reflecting one of the paradoxes inherent in the theory of rational choice (see Rapoport & Chammah, 1965). Here is the essence of the paradox. Two individuals are arrested as suspects in a serious crime. Each of them can plead either "guilty" or "not guilty." If both plead "guilty," each will be imprisoned for three years. If both plea "not guilty," each will be imprisoned for two years. If one of them pleads "guilty," and the second one "not guilty," then the first one will be sentenced for one year, and the second one for 50 years. These conditions are shown in the following matrix:

Table 5.2

	B pleas "not guilty"	**B** pleas "guilty"
A pleas "not guilty"	-2, -2	-50, -1
A pleas "guilty"	-1, -50	-3, -3

It seems that it would be advantageous for both of them to plead "not guilty", so that each would be sentenced to two years. But if one of them learns that the other decides to plead "not guilty", then it is better for him to plead "guilty" and receive only one year, although the other will be imprisoned for 50. Thus, to avoid the risk, the players ought to choose the strategy "guilty", but in this case each of them will be sentenced to three years instead of two (what they would get if they pleaded "not guilty"). These considerations are repeated continuously, and there is no obvious plan of rational behavior which would be unconditionally correct.

The model of the agent with reflexion allows us to take into consideration the moral aspect in addition to the utilitarian one. Imagine that these suspects have committed no crime and were arrested mistakenly. In this case, the alternative "plead guilty" means telling a lie and as such plays the role of the negative pole; the alternative "plead not guilty" is the truth and plays the role of the positive pole. It is easy to assume that the prospect of spending 50 years in prison is a catastrophe for each agent, in comparison with which terms of one, two or even three years look like acceptable outcomes. Therefore, the play-matrix given above corresponds to the following Boolean matrix:

Table 5.3

		B_3	
		1	0
x_3	1	1, 1	0, 1
	0	1, 0	1, 1

The Boolean values on the left and at the top indicate which strategies play the role of positive and negative poles for each player. The pair of values in the squares are evaluations of the outcomes: the first one for **A**, and the second one for **B**.

We will construct now function W for player **A**. He knows the rules of the game and understands that for his choice of the first strategy, (x_3=1), if the other also chooses the first strategy, (B_3=1), the outcome will be acceptable, (W=1), but if the other chooses the second strategy, (B_3=0), then the outcome for **A** will be disastrous, (W=0). For **A**'s choice of the second strategy, (x_3=0), the outcome will be acceptable, (W=1), independently of **B**'s choice. Therefore for **A**, relation between different strategies is as follows:

Table 5.4

x_3	B_3	W
1	1	1
1	0	0
0	1	1
0	0	1

From this table we find the analytical form of the function:

$$W = \overline{x}_3 + B_3. \tag{5.5}$$

In the case where no surprise takes place at the moment of choice, $a_1=a_2$, and the agent can be represented as

$$a_1 + \overline{a}_1(\overline{x}_3 + B_3) = A_1, \tag{5.6}$$

and after transformations:

$$a_1 + \overline{x}_3 + B_3 = A_1. \tag{5.7}$$

Let agent **A** tend to perform an intentional metachoice. This means that $x_3 \equiv A_1$, and we obtain the equation

$$a_1 + \overline{x}_3 + B_3 \equiv x_3. \tag{5.8}$$

It follows from (5.8) that when $a_1=0$ and $B_3=0$, identity $\overline{x}_3 \equiv x_3$ appears, so that (5.8) has no solutions, and the agent cannot produce a set of programs for his bipolar choice. This inability can be regarded as the agent's individual "experience" of the dilemma.

6. REFLEXIVE CONTROL OVER METACHOICE

Reflexive control is a deliberate attempt to influence on the agent's conscious and unconscious domains. Within the framework of the model of metachoice, we can single out three broad classes of reflexive control:

(1) Determining the agent's will.
(2) Determining the model of the situation.
(3) Determining the agent's value system.

There can also be combined forms of reflexive control which employ influences from different classes.

1. Determining the Agent's Will

Within this type of reflexive control, a controlling party forces the agent to choose one program from a set generated by the agent himself (Fig.6.1).

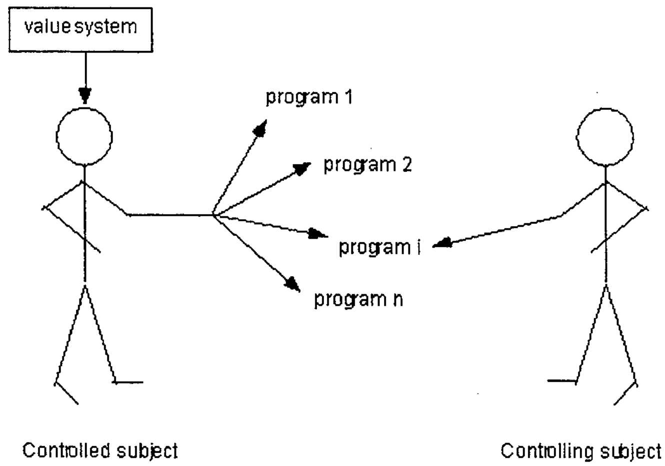


Fig.6.1. A controlling party determines the program to choose out of the set generated by the agent himself.

Consider the first basic example (section 4), in which the UN indicts a criminal syndicate only under the condition that both countries **A** and **B** file their complains. Imagine that the leadership of the syndicate succeeds in modeling the generation of the set of programs by country **A** in the same way we did, and received, as a result, a set consisting of two programs:

$$\psi_1 = a_1,$$

$$\psi_2 = a_1 + \bar{a}_2 B_3.$$

It is clear that when a_1=0 and a_2=1, equations $\psi_1 = 0$ and $\psi_2 = 0$ hold simultaneously. This means that the criminal syndicate can force country **A** to remain silent by choosing the following strategy: do not threaten during the first meeting (a_2=1); threaten during the second meeting (a_1=0).

Consider now a more realistic case, in which the leadership of the syndicate does not completely control the people conducting negotiations at the first meeting. Then the leadership cannot exclude the possibility that the threat has sounded, (a_2=0). If **A** chooses program $\psi_1 = a_1$, then a threat an hour prior to the UN session, (a_1=0), will accomplish the task. If **A**, however, chooses program $\psi_2 = a_1 + \bar{a}_2 B_3$, then following a threat made an hour prior to the UN session, **A**'s readiness to demand indictment will be $\psi_2' = B_3$; that is, **A** will make the decision which, from its point of view, **B** will make. Imagine further that the syndicate cannot influence **B**'s choice; it can still achieve its goal if, well before the first meeting the syndicate succeeds in persuading **A** that **B** will conceal its choice from **A**. This would incline **A** to choose plan $\psi_1 = a_1$, because this plan does not depend on **B**. In this case, **A**'s choice is in the syndicate's hands. A threat an hour before the UN session will incline **A**'s government to refrain from demanding indictment, regardless of **B**'s real decision. The chief principle of control over the adversary will consists in conveying to him a system of *argumentation*, which would lead him to the choice of plan advantageous to the controlling party. To conduct this kind of control, however, one needs information about the set of plans generated by the controlled agent. The $<x_1,x_2,M>$-model serves for deduction of this very set.

2. Determining the Model of the Situation

The essence of this type of reflexive control consists in determining not a single program, but a set of programs for the future actions. It can be achieved through constructing the special function $W(x_3,B_3)$ describing the situation (Fig.6.2).

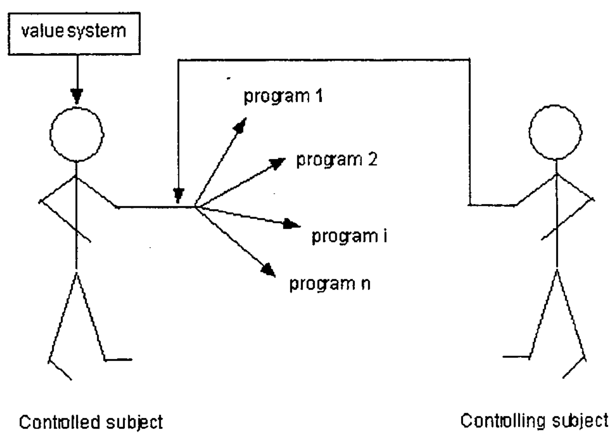


Fig.6.2. The controlling agent determines the set of programs generated by the controlled agent.

Let the goal of the criminal syndicate be the same as in the previous example: by threatening country *A* during the second meeting, to make it to refrain from demanding indictment. This can be achieved by substituting function $W(x_3,B_3)\equiv 0$ for function $W(x_3,B_3)=x_3B_3$. To accomplish this task, it is necessary to persuade country **A** that the UN will not indict the syndicate no matter what actions are taken by countries **A** and **B**. We see in Table 3.3 that in this case, the set of programs generated by **A** will consist of only one program, $\psi=a_1$, which can be completely controlled by the syndicate at the time of the second meeting.

3. Determining the Agent's Value System

The essence of this type of reflexive control consists of influencing **A**'s general criteria for the evaluation of actions and outcomes (Fig.6.3).

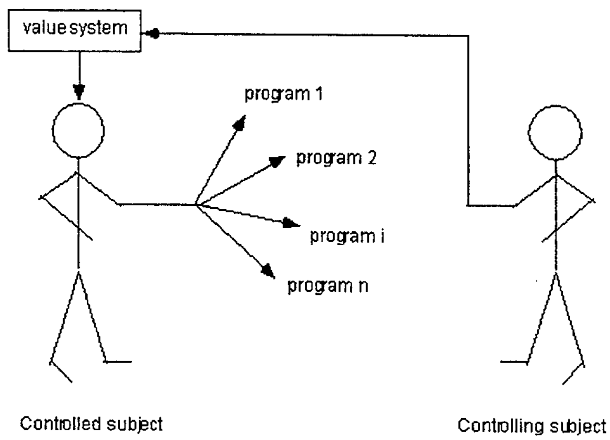


Fig.6.3. The controlling agent determines the value system of the controlled agent.

Let the syndicate have the goal of destroying the very procedure by which the government of country **A** reaches its decisions. The syndicate can achieve this by providing the government with an insoluble dilemma. It organizes a broad propaganda campaign promoting the idea of unity between countries **A** and **B** as more important than any UN resolution. In this way a positive evaluation will be given only to those outcomes in which countries **A** and **B** either both demand the indictment or both agree not to demand it. If such a campaign is successful, then function $W(x_3,B_3)$ will correspond to the following table:

Table 6.1

x_3	B_3	$W(x_3,B_3)$
1	1	1
1	0	0
0	1	0
0	0	1

It follows from this table that

$$W(x_3,B_3) = x_3B_3 + \overline{x}_3\overline{B}_3. \qquad (6.1)$$

Hence the intentional choice is described by the following equation:

$$a_1 + \bar{a}_2(x_3 B_3 + \bar{x}_3 \bar{B}_3) \equiv x_3. \tag{6.2}$$

This equation has no solution, since function (6.1) is a D-function (see Table 3.1). Therefore, **A**'s government faces an insoluble dilemma.

. . .

We see that modeling the process of metachoice allows us to analyze in detail the methods of informational influences on the process of strategic decision making.

7. A BOOLEAN-LINEAR MODEL OF THE AGENT

Consider a Boolean agent

$$A_1 = a_1 + \bar{a}_2 W(x_3, B_3),$$

where all variables take on values from the set $\{0,1\}$. Let variables a_1, a_2, x_3 and B_3 take on the Boolean values 1 and 0 independently of each other with the following probabilities of the appearance of 1:

$$|a_1| = x_1,$$
$$|a_2| = x_2,$$
$$|x_3| = x,$$
$$|B_3| = y.$$

To clarify the meaning of the values x_1, x_2, x, y, let the agent who is facing a choice between the poles experience multiple "jolts" which incline him toward choosing different poles. We presume that every jolt comes independently of the previous jolts. The probability of a "positive" jolt's appearance is constant and equal to x_1. Thus, x_1 is not only the probability, but also the frequency of microjolts toward the positive pole. Analogously, x_2 is the frequency of appearance of a microjolt toward the positive pole in the agent's mental image of the past; x is the frequency of the agent's intention to choose the positive pole, and y is the frequency with which the agent imagines his partner's choosing the positive pole.

Using these values we can find the probabilities that functions $W(x_3,B_3)$ and A_1 will take on the Boolean value of 1. Let

$$|W(x_3,B_3)| = M(x,y), \quad |A_1| = X_1. \tag{7.1}$$

The value of $M(x,y)$ is the frequency with which the agent assesses the situation positively, and X_1 is the frequency with which the agent's executive system is ready to choose the positive pole. The values of x_1 and x_2 will be called the *actual* and *expected pressures* toward the positive pole. Variables $1\text{-}X_1$, $1\text{-}M$, $1\text{-}x$, $1\text{-}y$, $1\text{-}x_1$, and $1\text{-}x_2$ are defined in a similar way with respect to the negative pole.

Direct computations demonstrate that

$$X_1 = x_1 + (1-x_1)(1-x_2)M(x,y). \tag{7.2}$$

The Boolean model of the situation $W(x_3,B_3)$ corresponds to the function $M(x,y)$, and Boolean models of the self (x_3) and of the other (B_3) correspond to variables x and y. It is easy to see that function $M(x,y)$ can be represented as bi-linear form:

$$M(x,y) = axy + bx + cy + d, \tag{7.3}$$

where a, b, c, and d are real numbers.

On the other hand, for the arithmetical values of $x,y \in \{0,1\}$ function $M=M(x,y)$ "models" the Boolean function $W(x_3,B_3)$, i.e., there is a one-to-one correspondence between the Boolean values 1 and 0 of variables x_3, B_3, W and the arithmetical values 0 and 1 of variables x, y, M. Therefore, to determine a function $M(x,y)$, we have to vary the arithmetical values 1 and 0 of variables x and y and, using function $W(x_3,B_3)$, find the value of function $M(x,y)$ for each combination. As a result we obtain a system of linear equations:

$$a \cdot 1 \cdot 1 + b \cdot 1 + c \cdot 1 + d = \alpha_1$$

$$a \cdot 1 \cdot 0 + b \cdot 1 + c \cdot 0 + d = \alpha_2$$

$$a \cdot 0 \cdot 1 + b \cdot 0 + c \cdot 1 + d = \alpha_3$$

$$a \cdot 0 \cdot 0 + b \cdot 0 + c \cdot 0 + d = \alpha_4 \,,$$

or

$$a+b+c+d=\alpha_1$$

$$b+d=\alpha_2$$

$$c+d=\alpha_3$$

$$d=\alpha_4,$$

where α_i represents integer (1 or 0) values of $M(x,y)$. By solving this system we find the coefficients a, b, c, d, and correspondingly, the function $M(x,y)$. Table 7.1 shows all 16 functions $M(x,y)$.

Table 7.1. A set of functions $M_i(x,y)=axy+bx+cy+d$

i	$W_i(x_3,B_3)$	$M_i(x,y)$	$a,\ b,\ c,\ d$
1	0	0	0, 0, 0, 0
2	x_3	x	0, 1, 0, 0
3	$\bar{x}_3$	$1-x$	0, -1, 0, 1
4	B_3	y	0, 0, 1, 0
5	$\bar{B}_3$	$1-y$	0, 0, -1, 1
6	x_3B_3	xy	1, 0, 0, 0
7	$\bar{x}_3B_3$	$(1-x)y$	-1, 0, 1, 0
8	$x_3\bar{B}_3$	$x(1-y)$	-1, 1, 0, 0
9	$\bar{x}_3\bar{B}_3$	$1-x-y+xy$	1, -1, -1, 1
10	x_3+B_3	$x+y-xy$	-1, 1, 1, 0
11	$x_3+\bar{B}_3$	$1-y+xy$	1, 0, -1, 1
12	$\bar{x}_3+B_3$	$1-x+xy$	1, -1, 0, 1
13	$\bar{x}_3+\bar{B}_3$	$1-xy$	-1, 0, 0, 1
14	$\bar{x}_3B_3+x_3\bar{B}_3$	$x+y-2xy$	-2, 1, 1, 0
15	$x_3B_3+\bar{x}_3\bar{B}_3$	$1-x-y+2xy$	2, -1, -1, 1
16	1	1	0, 0, 0, 1

Function $M(x,y)$ is linear on x, so that (7.2) can be represented as

$$A(y)x+B(y)=X_1, \tag{7.4}$$

where

$$A(y)=(1-x_1)(1-x_2)(ay+b), \tag{7.5}$$

$$B(y)=x_1+(1-x_1)(1-x_2)(cy+d). \tag{7.6}$$

The values of the parameters a, b, c and d are given in Table 7.1. Since function (7.4) is linear on x, we call this model *Boolean-linear.*

Let us compare a Boolean-linear model of the agent with the general scheme $|readiness|=F_D(|intention|)$.

(1) The following function plays the role of F_D:

$$X_1=x_1+(1-x_1)(1-x_2)M(x,y)$$

(2) $|readiness|$ and $|intention|$ are numbers from the interval [0,1], where $|readiness|$ is the value of variable X_1, and $|intention|$ that of x.

(3) Parameter D is the set {function $M(x,y)$, numbers $x_1,x_2,y \in [0,1]$} where $M(x,y)$ is bi-linear form (7.3) with the values of parameters a, b, c, d given in Table 7.1.

It is important to emphasize that, in the framework of the Boolean-linear model, we consider an intentional choice to be the only bipolar one; in other words, the values of X_1 and x are numbers, not functions.

Our next step consists of extending the idea of an intentional choice from the Boolean model to the Boolean-linear one. We call the agent's choice *intentional*, if his readiness X_1 to choose the positive pole is equal to his intention, x, to do so. The agent capable of making an intentional choice corresponds to the following equation:

$$x_1+(1-x_1)(1-x_2)M(x,y)=x, \tag{7.7}$$

or in a standard form

$$A(y)x+B(y)=x, \tag{7.8}$$

where $A(y)$ and $B(y)$ are determined by equations (7.5) and (7.6). Therefore, an intentional choice corresponds to a transformation of the type $Ax+B$, and the probability of choice corresponds to a fixed point of this transformation. Such a transformation, in general, has the following properties:

(a) $Ax+B$ has no single fixed point if $A=1$ and $B\neq 0$

(b) $Ax+B$ has only one fixed point if $A\neq 1$

(c) for $Ax+B$, every point from the domain of x is a fixed one, if $A=1$ and $B=0$.

The analysis of all transformations $A(y)x+B(y)$ demonstrates that case (a) is impossible, and cases (b) and (c) are possible. The latter corresponds to the agent's state in which his any intention, x, automatically turns into readiness X_1. According to the definition introduced in Section 2, in this case the agent has *freedom of choice*.

Table 7.2 presents the solutions of equations (7.7) for all 16 functions $M(x,y)$ given in Table 7.1. The corresponding functions $W(x_3,B_3)$ are also presented there. Each solution can be represented as a function of variable y in which x_1 and x_2 are parameters:

$$x=P_{x1,x2}(y). \tag{7.9}$$

Not every function (7.9) is single-valued, but for every pair of values x_1, x_2, every function x has a graph that is continuous in every point $y\in[0,1]$. Any case for which function (7.9) is not single-valued corresponds to the appearance of the agents' freedom of choice.

Table 7.2

Solutions of the equations of the type $A(y)x+B(y)=x$.

1.

<table>
<tr><td>x_3</td><td>B_3</td><td>$W_1=0$</td><td rowspan="2">$M_1=0$</td></tr>
<tr><td>1</td><td>1</td><td>0</td></tr>
<tr><td>1</td><td>0</td><td>0</td><td rowspan="3">$x=x_1$</td></tr>
<tr><td>0</td><td>1</td><td>0</td></tr>
<tr><td>0</td><td>0</td><td>0</td></tr>
</table>

2.

<table>
<tr><td>x_3</td><td>B_3</td><td>$W_2=x_3$</td><td rowspan="2">$M_2=x$</td></tr>
<tr><td>1</td><td>1</td><td>1</td></tr>
<tr><td>1</td><td>0</td><td>1</td><td rowspan="3">$x=\begin{cases}\dfrac{x_1}{x_1+x_2-x_1x_2}, & \text{if } x_1+x_2>0\\ \text{any}\in[0,1], & \text{if } x_1+x_2=0\end{cases}$</td></tr>
<tr><td>0</td><td>1</td><td>0</td></tr>
<tr><td>0</td><td>0</td><td>0</td></tr>
</table>

3.

<table>
<tr><td>x_3</td><td>B_3</td><td>$W_3=\bar{x}_3$</td><td rowspan="2">$M_3=1-x$</td></tr>
<tr><td>1</td><td>1</td><td>0</td></tr>
<tr><td>1</td><td>0</td><td>0</td><td rowspan="3">$x=\dfrac{1-x_2+x_1x_2}{1+(1-x_1)(1-x_2)}$</td></tr>
<tr><td>0</td><td>1</td><td>1</td></tr>
<tr><td>0</td><td>0</td><td>1</td></tr>
</table>

4.

x_3	B_3	$W_4 = B_3$	$M_4 = y$
1	1	1	
1	0	0	$x = x_1 + (1-x_1)(1-x_2)y$
0	1	1	
0	0	0	

5.

x_3	B_3	$W_5 = \overline{B}_3$	$M_5 = 1-y$
1	1	0	
1	0	1	$x = x_1 + (1-x_1)(1-x_2)(1-y)$
0	1	0	
0	0	1	

6.

x_3	B_3	$W_6 = x_3 B_3$	$M_6 = xy$
1	1	1	
1	0	0	$x = \begin{cases} \dfrac{x_1}{1-(1-x_1)(1-x_2)y}, & \text{if not}(y=1 \ \& \ x_1+x_2=0) \\ \text{any} \in [0,1], & \text{if } (y=1 \ \& \ x_1+x_2=0) \end{cases}$
0	1	0	
0	0	0	

7.

x_3	B_3	$W_7 = \bar{x}_3 B_3$	$M_7 = (1-x)y$
1	1	0	
1	0	0	$x = \dfrac{x_1 + (1-x_1)(1-x_2)y}{1 + (1-x_1)(1-x_2)y}$
0	1	1	
0	0	0	

8.

<table>
<tr><td>x_3</td><td>B_3</td><td>$W_8 = x_3\overline{B}_3$</td><td rowspan="2">$M_8=x(1-y)$</td></tr>
<tr><td>1</td><td>1</td><td>0</td></tr>
<tr><td>1</td><td>0</td><td>1</td><td rowspan="3">$x = \begin{cases} \dfrac{x_1}{(x_1+x_2-x_1x_2)+(1-x_1)(1-x_2)y}, & \text{if not}(y=0 \ \&\ x_1+x_2=0) \\ \text{any} \in [0,1], & \text{if } (y=0 \ \&\ x_1+x_2=0) \end{cases}$</td></tr>
<tr><td>0</td><td>1</td><td>0</td></tr>
<tr><td>0</td><td>0</td><td>0</td></tr>
</table>

9.

<table>
<tr><td>x_3</td><td>B_3</td><td>$W_9 = \overline{x}_3\overline{B}_3$</td><td rowspan="2">$M_9=(1-x)(1-y)$</td></tr>
<tr><td>1</td><td>1</td><td>0</td></tr>
<tr><td>1</td><td>0</td><td>0</td><td rowspan="3">$x=\dfrac{1-x_2(1-x_1)-(1-x_1)(1-x_2)y}{1+(1-x_1)(1-x_2)(1-y)}$</td></tr>
<tr><td>0</td><td>1</td><td>0</td></tr>
<tr><td>0</td><td>0</td><td>1</td></tr>
</table>

10.

<table>
<tr><td>x_3</td><td>B_3</td><td>$W_{10}=x_3+B_3$</td><td rowspan="2">$M_{10}=x+y-xy$</td></tr>
<tr><td>1</td><td>1</td><td>1</td></tr>
<tr><td>1</td><td>0</td><td>1</td><td rowspan="3">$x = \begin{cases} \dfrac{x_1+(1-x_1)(1-x_2)y}{1-(1-x_1)(1-x_2)+(1-x_1)(1-x_2)y}, & \text{if not}(y=0 \ \&\ x_1+x_2=0) \\ \text{any} \in [0,1], & \text{if } (y=0 \ \&\ x_1+x_2=0) \end{cases}$</td></tr>
<tr><td>0</td><td>1</td><td>1</td></tr>
<tr><td>0</td><td>0</td><td>0</td></tr>
</table>

11.

x_3	B_3	$W_{11}= x_3+\overline{B}_3$	M_{11}=1-y-xy
1	1	1	
1	0	1	$x = \begin{cases} \dfrac{x_1+(1-x_1)(1-x_2)(1-y)}{1-(1-x_1)(1-x_2)y}, & \text{if not}(y=1\ \&\ x_1+x_2=0) \\ \text{any}\in[0,1], & \text{if } (y=1\ \&\ x_1+x_2=0) \end{cases}$
0	1	0	
0	0	1	

12.

x_3	B_3	$W_{12}= \overline{x}_3+B_3$	M_{12}=1-x+xy
1	1	1	
1	0	0	$x=\dfrac{1-x_2+x_1x_2}{1+(1-x_1)(1-x_2)(1-y)}$
0	1	1	
0	0	1	

13.

x_3	B_3	$W_{13}= \overline{x}_3+\overline{B}_3$	M_{13}=1-xy
1	1	0	
1	0	1	$x=\dfrac{1-x_2+x_1x_2}{1+(1-x_1)(1-x_2)y}$
0	1	1	
0	0	1	

14.

<table>
<tr><td>x_3</td><td>B_3</td><td>$W_{14}=$ $\bar{x}_3 B_3 + x_3 \bar{B}_3$</td><td rowspan="2">$M_{14}=x+y-2xy$</td></tr>
<tr><td>1</td><td>1</td><td>0</td></tr>
<tr><td>1</td><td>0</td><td>1</td><td rowspan="3">$x = \begin{cases} \dfrac{x_1+(1-x_1)(1-x_2)y}{(x_1+x_2-x_1x_2)+2(1-x_1)(1-x_2)y}, & \text{if not}(y=0 \ \&\ x_1+x_2=0) \\ \text{any} \in [0,1], & \text{if } (y=0 \ \&\ x_1+x_2=0) \end{cases}$</td></tr>
<tr><td>0</td><td>1</td><td>1</td></tr>
<tr><td>0</td><td>0</td><td>0</td></tr>
</table>

15.

<table>
<tr><td>x_3</td><td>B_3</td><td>$W_{15}=$ $x_3 B_3 + \bar{x}_3 \bar{B}_3$</td><td rowspan="2">$M_{15}=1-x-y+2xy$</td></tr>
<tr><td>1</td><td>1</td><td>1</td></tr>
<tr><td>1</td><td>0</td><td>0</td><td rowspan="3">$x = \begin{cases} \dfrac{1-x_2(1-x_1)-(1-x_1)(1-x_2)y}{1+(1-x_1)(1-x_2)(1-2y)}, & \text{if not}(y=1 \ \&\ x_1+x_2=0) \\ \text{any} \in [0,1], & \text{if } (y=1 \ \&\ x_1+x_2=0) \end{cases}$</td></tr>
<tr><td>0</td><td>1</td><td>0</td></tr>
<tr><td>0</td><td>0</td><td>1</td></tr>
</table>

16.

<table>
<tr><td>x_3</td><td>B_3</td><td>$W_{16}=1$</td><td rowspan="2">$M_{16}=1$</td></tr>
<tr><td>1</td><td>1</td><td>1</td></tr>
<tr><td>1</td><td>0</td><td>1</td><td rowspan="3">$x = 1-x_2+x_1x_2$</td></tr>
<tr><td>0</td><td>1</td><td>1</td></tr>
<tr><td>0</td><td>0</td><td>1</td></tr>
</table>

8. EXAMPLES OF USING BOOLEAN-LINEAR MODEL OF THE AGENT

We return now to the first example from section 4. Let **A**'s government be unable to develop a program of its actions in advance. Suppose that, during the first meeting, the representatives of the criminal syndicate threatened country **A**, so x_2=0; during the second meeting, however, they softened their position, so that x_1=(1/4). In accordance with the example examined in section 4, country **A** has a model of the situation represented by the Boolean function

$$W(x_3,B_3)=x_3B_3. \tag{8.1}$$

From Table 7.2 we find that Boolean function (8.1) fits situation 6, which corresponds to the following continuous function

$$M(x,y)=xy. \tag{8.2}$$

Therefore, **A**'s intentional choice is described by the following equation:

$$x_1+(1-x_1)(1-x_2)xy=x. \tag{8.3}$$

A general form of the solution for this equation can be found in Table 7.2:

$$x = \begin{cases} \dfrac{x_1}{1-(1-x_1)(1-x_2)y}, & \text{if not}(y=1 \ \& \ x_1+x_2=0) \\ \text{any}\in[0,1], & \text{if } (y=1 \ \& \ x_1+x_2=0). \end{cases} \tag{8.4}$$

By substituting values x_1=(1/4) and x_2=0 into this equation, we obtain

$$x = \frac{1}{4-3y}. \tag{8.5}$$

The above value of x can be interpreted as the probability with which country **A** will speak up at the UN session. We see that this probability is a function of the decision made by country **B**, from **A**'s point of view.

Let us consider now the case where country A evaluates positively only those outcomes in which choices of **A** and **B** coincide. As it was shown before (see (6.1)), in this case

$$W(x_3,B_3)=x_3B_3+\overline{x}_3\overline{B}_3. \tag{8.6}$$

By looking at Table 7.2, we find that this is situation 15; it corresponds to the function

$$M(x,y)=1-x-y+2xy, \tag{8.7}$$

and **A**'s intentional choice corresponds to the equation

$$x_1+(1-x_1)(1-x_2)(1-x-y+2xy)=x. \tag{8.8}$$

A general form of solution of this equation is found in Table 7.2:

$$x = \begin{cases} \dfrac{1-x_2(1-x_1)-(1-x_1)(1-x_2)y}{1+(1-x_1)(1-x_2)(1-2y)}, & \text{if not}(y=1 \ \& \ x_1+x_2=0) \\ \text{any}\in[0,1], & \text{if } (y=1 \ \& \ x_1+x_2=0) \end{cases} \tag{8.9}$$

By substituting of values x_1=(1/4) and x_2=0, we obtain

$$x = \frac{4-3y}{7-6y}. \tag{8.10}$$

Let us now suppose that in the framework of the examples considered above, the full-scale threats were made during both the first meeting and the second one, that is, x_1=0 and x_2=0. By substituting these values into (8.4), we obtain

$$\mathrm{x} = \begin{cases} 0, \text{ if } y\neq 1 \\ \text{any}\in[0,1], \text{ if } y=1. \end{cases} \tag{8.11}$$

It follows from (8.11) that if country **A** is not completely sure about **B**'s willingness to speak out, **A** will refrain from demanding indictment. If **A** has not doubt about **B**'s actions, **A**'s choice cannot be determined by an external party: country **A** has the ability to make a free choice.

Substitute values of x_1=0 and x_2=0 into (8.9):

$$\mathrm{x} = \begin{cases} 1/2, & \text{if } y\neq 1 \\ \text{any}\in[0,1], & \text{if } y=1. \end{cases} \tag{8.12}$$

It follows from (8.12) that if country **A** has any doubt in B's speaking out, **A** will choose either alternative with equal probabilities. If **A** has no doubts, it acquires the ability to make a free choice. It is important to emphasize the difference between the cases with x=1/2 and that with x=any$\in$[0,1]. In the first case, we know the probability and can use it in subsequent computations. In the second case, we have no information at all about the probability of the agent's choosing the alternatives. The difference between these two cases is essential for planning reflexive control (see section 12).

9. REFLEXIVE CONTROL OVER BIPOLAR CHOICE

A Boolean-linear model describes the agent's state immediately before the act of bipolar choice. By this moment the agent usually has a stable value system, a model of the situation, $M(x,y)$, and some anticipation of pressure from the external world. Thus, for a party using the Boolean-linear model to develop controlling influences on the enemy agent, it is expedient to concentrate its attention on the values

$$x, x_1, y.$$

Variable x represents the agent's will or inner intention; parameter x_1 shows the agent's unconscious perception of the pressure toward the positive pole, and parameter y stands for the cognizant image of the other agent's actions.

In intentional bipolar choice, a manipulation with variable x is possible only in those cases, where the agent can make a free choice. In all other cases, the value of x is determined unambiguously by the values of x_1, x_2, and y. All the situations and conditions under which the manipulation with x-value is possible are given in Table 9.1 (based on Table 7.2).

Table 9.1. A set of conditions for a direct manipulation with variable x

i	W_i	M_i	y	x_1	x_2
2	x_3	x	any$\in[0,1]$	0	0
6	x_3B_3	xy	1	0	0
8	$x_3\overline{B}_3$	$x(1-y)$	0	0	0
10	x_3+B_3	$x+y-xy$	0	0	0
11	$x_3+\overline{B}_3$	$1-y+xy$	1	0	0
14	$\overline{x}_3B_3+x_3\overline{B}_3$	$x+y-2xy$	0	0	0
15	$x_3B_3+\overline{x}_3\overline{B}_3$	$1-x-y+2xy$	1	0	0

We have to note that for all 16 equations of intentional choice, readiness $x\equiv 1$, when $x_1=1$, and $x\equiv 0$ when $x_1=0$ and $x_2=1$. Only for some equations is readiness $x=0$, when $x_1=0$ and $x_2=0$ (under conditions that y takes on a special value y^*). They are those very functions for which it is appropriate to make a threat immediately before the controlled party is to make its decision (if it is known that $y=y^*$). Only for function $W(x, y) \equiv 0$, a last-minute threat is unconditionally effective.

Further we examine a set of illustrative problems and their solutions related to finding controlling influences over an enemy.

Problem 1. The syndicate knows that, during the first meeting, its representatives threatened country **A**; for country **A** the only positive outcome is the case when both countries **A** and **B** enunciate their demand. The syndicate's goal is to keep **A** silent.

(a) Is it necessary to threaten during the second meeting?

(b) Is it possible to influence **A**'s intention?

(c) Is it expedient to influence **A**'s cognizant image of **B**?

Solution. First we construct a model of agent **A**. A general form of the equation is

$$x=x_1+(1-x_1)(1-x_2)M(x,y). \tag{9.1}$$

From the fact that there was a threat during the first meeting, we obtain $x_2=0$. Since a positive outcome for **A** is for **A** and **B** express their demand together, $W=x_3B_3=W_6$, and $M=xy=M_6$. As a result, equation (9.1) acquires the form

$$x=x_1+(1-x_1)xy. \tag{9.2}$$

We solve this equation and obtain

$$x=\frac{x_1}{1-(1-x_1)y}. \tag{9.3}$$

(a) A threat during the second meeting means $x_1=0$. When $x_1=0$, the value $x=0$ appears if $y<1$. When $y=1$, the agent has freedom of choice. The absence of a threat during the second meeting means $x_1=1$ which entails $x=1$. Thus a threat during the second meeting is necessary.

(b) Influence on **A**'s intention is possible only if **A** has freedom of choice, which appears when $x_1=0$ and $y=1$. Thus it is possible to influence **A**'s intention if (1) **A** is convinced that **B** will speak up ($y=1$) and (2) there is a threat made during the second meeting ($x_1=0$).

(c) If $x_1=0$, the value $x=0$ appears only if $y<1$. Thus, it is expedient to influence **A**'s cognizant image of **B**: this will ensure that **A** is not certain of **B**'s actions (in demanding the indictment).

Problem 2. The syndicate knows that **A** regards positively the outcome where both **A** and **B** demand the indictment and negatively the outcome where **B** refrains from making this demand. The evaluation of the outcome in which **A** remains silent and **B** asserts its demand is unknown. The syndicate threatened country **A** during the first meeting and plans to threaten again during their second meeting. How can the syndicate conduct reflexive control?

Solution. This description does not allow us to restore a function $W(x_3B_3)$ unambiguously. Two functions, $W_a(x_3,B_3)$ and $W_b(x_3,B_3)$, meet the description (Table 9.2).

Table 9.2

x	y	$W_p(x_3,B_3)$	$W_q(x_3,B_3)$
1	1	1	1
1	0	0	0
0	1	1	0
0	0	0	0

It follows from this table that $W_p=y$ and $W_q=xy$; that is, $p=4$ and $q=6$. Turning to Table 7.2, we find two functions satisfying the conditions described ($x_1=x_2=0$):

$$x=y, \tag{9.4}$$

$$x = \begin{cases} 0, \text{ if } y<1 \\ \text{any} \in [0,1], \text{ if } y=1. \end{cases} \tag{9.5}$$

A comparison of (9.4) and (9.5) shows that the syndicate must try to persuade **A** that **B** will refrain from demanding the indictment, i.e., $y=0$.

10. IMITATION OF ANOTHER AGENT

In the Boolean-linear model of the agent, the solution for an intentional choice has the form $x=f(x_1,x_2,y)$. This solution depends on a cognizant model of the other agent represented by the variable y. In many cases, however, the agent does not have any real information about the choice his partner or adversary may make. Psychological analysis of this problem shows that in such situations people tend to "role-play," as if they occupied two separate personalities: the self and the other. It is necessary to distinguish reflexion from this king of role-playing. Reflexion is mental modeling of the self and the other. Playing the role of the other, which we call *imitation* (Lefebvre, 1999a), employs the entire self for modeling the other. While attempting to penetrate into the adversary's thoughts, the agent tries not only to think like the adversary, but also to move like him, to speak with his intonations, to adopt facial expressions characteristic of him. In previous work (Lefebvre, 1999a) we demonstrated that an agent imitating another agent can be represented by the following system of equations:

$$\left.\begin{array}{l} x_1+(1-x_1)(1-y)M_x(x,y)=x \\ y_1+(1-y_1)(1-x)M_y(y,x)=y \end{array}\right\}, \qquad (10.1)$$

where the first equation corresponds to the agent himself and the second to his role-playing of the second agent. Let us note that when $M_x(x,y)$ and $M_y(x,y)$ are bi-linear functions from Table 7.1, system (10.1) consists of two bi-linear equations:

$$\left.\begin{array}{l} a_1xy+a_2x+a_3y+a_4=0 \\ b_1xy+b_2x+b_3y+b_4=0 \end{array}\right\}. \qquad (10.2)$$

It has been demonstrated earlier (Lefebvre, 1999a) that system (10.2) always has at least one solution ($x=x'$, $y=y'$) such that $x',y'\in[0,1]$. In addition, for any $y\in[0,1]$, the first equation has a root on x, and analogously, for any $x\in[0,1]$, the second equation has a root on y.

The idea of modeling imitation of the other's decisions by system (10.1) is based on the assumption that the agent *himself* (A) tries to reflect correctly the state of the other's (B's) readiness, who in his turn, correctly reflects the state of A's readiness.

We will describe now a procedure of forming the state of readiness in agent A.

(1) If only one value $y^*\in[0,1]$ satisfies system (10.2), i.e., a set of solution has the form $(x=x', y=y^*)$, where $x'\in S\subset[0,1]$, then A's readiness, x, can be equal to any $x'\in S$.

(2) If condition (1) does not hold, but there are such $x^*\in[0,1]$ for which

$$x_1+(1-x_1)(1-x_2)M(x^*,y)\equiv x^*, \qquad (10.3)$$

where y is any$\in[0,1]$, then A's readiness, x, can be equal to x^*.

(3) If neither condition described above holds, then A's readiness, x, cannot be established.

Point (1) corresponds to the case in which A can predict B's state of readiness. Point (2) describes the case when A cannot predict B's state unambiguously, but he can move to a state which does not depend on B's state. Point (3) corresponds to the case in which A can neither predict B's state, nor move to the state where he does not depend on B.

Let us consider three examples. First, $W_x=(1-x)y$, $x_1=x_2=0$ and $W_y=1-y$, $y_1=y_2=0$. We obtain the following system:

$$\left.\begin{array}{l} (1-x)y=x \\ 1-y=y \end{array}\right\}. \qquad (10.4)$$

System (10.4) has only one solution (x=1/3, y=1/2). By virtue of point (1), agent A moves into state x=1/3.

Second, $W_x=xy$, $x_1=x_2=0$ and $W_y=x$, $y_1=y_2=0$.

$$\left.\begin{array}{l} xy=x \\ x=y \end{array}\right\} . \tag{10.5}$$

System (10.5) has two solutions: (x=0, y=0) and (x=1, y=1). The condition of point (1) is not satisfied, but that of point (2) is, since at x=0 the first equation in (10.5) turns into $0y \equiv 0$, which does not depend on y; therefore, agent A moves into state x=0.

Third, $W_x=y$, $x_1=x_2=0$ and $W_y=x$, $y_1=y_2=0$.

$$\left.\begin{array}{l} y=x \\ x=y \end{array}\right\} . \tag{10.6}$$

System (10.6) has solutions of the type ($x=\alpha$, $y=\alpha$), where α are arbitrary numbers from the interval [0,1]. The conditions for the points (1) and (2) are not satisfied, so that A cannot establish a state of readiness. In this case A may stimulate B to move into any fixed state $y\in[0,1]$, only with the purpose of letting his own cognitive system form a certain value of x based on the information about y.

11. USING A METHOD OF REFLEXIVE GAMES TO FIND VALUES OF x_1, x_2, y_1, y_2

Classical game theory has from the very beginning had two interpretations. First of all, it was regarded as a *normative* theory designed to help people to make *optimal* decisions. Second, it was understood as a theory *describing* real human choice in situations of conflict. The second interpretation was based on the assumption that a human being is essentially rational and strives for optimal outcomes. Classical game theory continues to play the normative role with reasonable success, but as a descriptive theory it has not achieved its intended purpose, especially where individual choice is concerned (Leinfellner & Kohler, 1998). We believe that one of the main reasons for this is its privileging of the utilitarian over the deontological aspect of choice in polarization of the alternatives, despite the fact that real human choices involve both aspects.

We have earlier demonstrated the possibility of constructing a more general descriptive theory based on the Boolean-linear model and combining these two aspects (Lefebvre, 1999a). The main idea consists in using equations (10.1), where the values of x_1, x_2, y_1, and y_2 are the mixed strategy values.

We begin with constructing two non-zero-sum-game matrices using information on real situation. Both matrices are constructed from player A's point of view.

$$
\begin{array}{cc}
 & B \\
A & \begin{array}{|c|c|}\hline a^1_{11}, b^1_{11} & a^1_{12}, b^1_{12} \\ \hline a^1_{21}, b^1_{21} & a^1_{22}, b^1_{22} \\ \hline\end{array}
\end{array}
\qquad
\begin{array}{cc}
 & B \\
A & \begin{array}{|c|c|}\hline a^2_{11}, b^2_{11} & a^2_{12}, b^2_{12} \\ \hline a^2_{21}, b^2_{21} & a^2_{22}, b^2_{22} \\ \hline\end{array}
\end{array}
$$

The first matrix shows the utility-measures of the outcomes at the moment of choice; the second matrix shows the expected utility-measures, which were formed earlier based on previous experience. Then the modeling pursues the following scheme.

1. Each matrix is reconstructed into two matrices showing the utility-measures for each player separately and considered as the play matrix for a zero-sum game.

$$
\begin{array}{cc}
 & B \\
A & \begin{array}{|c|c|}\hline a^1_{11} & a^1_{12} \\ \hline a^1_{21} & a^1_{22} \\ \hline\end{array} \\
 & \text{(a)}
\end{array}
\quad
\begin{array}{cc}
 & B \\
A & \begin{array}{|c|c|}\hline b^1_{11} & b^1_{12} \\ \hline b^1_{21} & b^1_{22} \\ \hline\end{array} \\
 & \text{(b)}
\end{array}
\quad
\begin{array}{cc}
 & B \\
A & \begin{array}{|c|c|}\hline a^2_{11} & a^2_{12} \\ \hline a^2_{21} & a^2_{22} \\ \hline\end{array} \\
 & \text{(c)}
\end{array}
\quad
\begin{array}{cc}
 & B \\
A & \begin{array}{|c|c|}\hline b^2_{11} & b^2_{12} \\ \hline b^2_{21} & b^2_{22} \\ \hline\end{array} \\
 & \text{(d)}
\end{array}
$$

A's optimal mixed strategies, $(p_1, 1-p_1)$ and $(p_2, 1-p_2)$, are found from matrices (a) and (c); B's optimal mixed strategies (from A's point of view), $(q_1, 1-q_1)$ and $(q_2, 1-q_2)$, are found from matrices (b) and (d).

2. Using information about real situation, we determine which of the two pure strategies for each player is associated with the positive pole (we assume that p_1, p_2, and q_1, q_2, correspond to the positive poles).

3. Functions $M_x(x,y)$ and $M_y(y,x)$ are constructed based on the information about the real situation.

4. The system of equations is written down as follows:

$$x_1 + (1-x_1)(1-x_2)M_x(x,y) = x$$

$$y_1 + (1-y_1)(1-y_2)M_y(y,x) = y.$$

5. This system is analyzed in accordance with the rules described in section 10, the result of which is a prognosis about A's state of readiness.

Let us consider an example. Each country regards as positive only those outcomes where the other country asserts its demand. After the first meeting with the criminal syndicate representatives, a month prior to the UN session, country A has the following play-matrix describing its potential losses (in millions of dollars):

		B demand	B no demand
A	demand	$a^2_{11}=-1$ $b^2_{11}=-1$	$a^2_{12}=-2$ $b^2_{12}=-3$
A	no demand	$a^2_{21}=-3$ $b^2_{21}=-4$	$a^2_{22}=-1$ $b^2_{22}=0$

After the second meeting, the matrix changed to

		B demand	B no demand
A	demand	$a^2_{11}=-4$ $b^2_{11}=-2$	$a^2_{12}=-4$ $b^2_{12}=-3$
A	no demand	$a^2_{21}=-2$ $b^2_{21}=-3$	$a^2_{22}=-2$ $b^2_{22}=-1$

Problem: To develop a prognosis for country A's choice.
1. Construct matrices (a), (b), (c), and (d):

$$
A\ \overset{B}{\begin{array}{|c|c|}\hline -4 & -4 \\ \hline -2 & -2 \\ \hline\end{array}}
\qquad
A\ \overset{B}{\begin{array}{|c|c|}\hline -2 & -3 \\ \hline -3 & -1 \\ \hline\end{array}}
\qquad
A\ \overset{B}{\begin{array}{|c|c|}\hline -1 & -2 \\ \hline -3 & -1 \\ \hline\end{array}}
\qquad
A\ \overset{B}{\begin{array}{|c|c|}\hline -1 & -3 \\ \hline -4 & 0 \\ \hline\end{array}}
$$

(a) (b) (c) (d)

From matrices (a) and (c) we find optimal mixed strategies for player A; the following numbers corresponds to the upper rows:

$$p_1=0, \quad p_2=2/3,$$

and from matrices (b) and (d) for player B; the following numbers corresponds to the left columns:

$$q_1=2/3, \quad q_2=1/2.$$

2. The strategies connected with the demand for indictment of the syndicate are positive for each country, A and B, so that

$$x_1=0, \qquad x_2=2/3$$
$$y_1=2/3, \qquad y_2=1/2.$$

3. Each country regards as positive that outcome where only the other country asserts its demand, so that

$$W_x(x,y) = y, \quad W_y(y,x) = x.$$

4. Let us write a system of equation:

$$0 + (1-0)(1-\frac{2}{3})y = x$$

$$\frac{2}{3} + (1-\frac{2}{3})(1-\frac{1}{2})x = y,$$

or

$$\frac{1}{3}y = x$$

$$\frac{2}{3} + \frac{1}{6}x = y.$$

5. This system has only one solution:

$$x = \frac{4}{17} \approx 0.24, \quad y = \frac{12}{17} \approx 0.71.$$

Therefore, From A's point of view, B will express its demand with a probability of 0.71, and A will do so with a probability of 0.24.

12. REALIZATION OF A CHOICE

In our previous report (Lefebvre, 1999b), we introduced the values $P_{\max}$ and $P_{\min}$ limiting the area in which the agent's cognitive system is capable of turning readiness into choice. It is necessary to emphasize that these limit values cannot be computed with the help of the formal reflexive model, they require additional theoretical investigation. A brief glance at the history of making important decisions reveals a peculiar difficulty: people do not easily accept the very situation of choice. In our terminology we can say that the readiness to choose is not established. For some people, this phenomenon occurs when the world exercises intense pressure toward one of the alternatives, and for others when the pressure is approximately equal toward each alternative. We will call the former agents I, and the later agents II. To clarify the formal scheme to be introduced presently, let us consider two statements, which *metaphorically* describe agents I and II:

Agent I:

In making a choice I will lose something. Before I make my choice I potentially possess both alternatives; after my choice I have only one. The very fact that in the future I have the opportunity to choose is valuable for me, I don't want to loose it. Thus I won't make any choice for the time being..

Agent II:

A state of uncertainty is difficult for me to tolerate. In this state I don't possess either of the alternatives. I need to avoid being in this state, and if I am already in it, I'd better escape from it as soon as possible by choosing one of the alternatives.

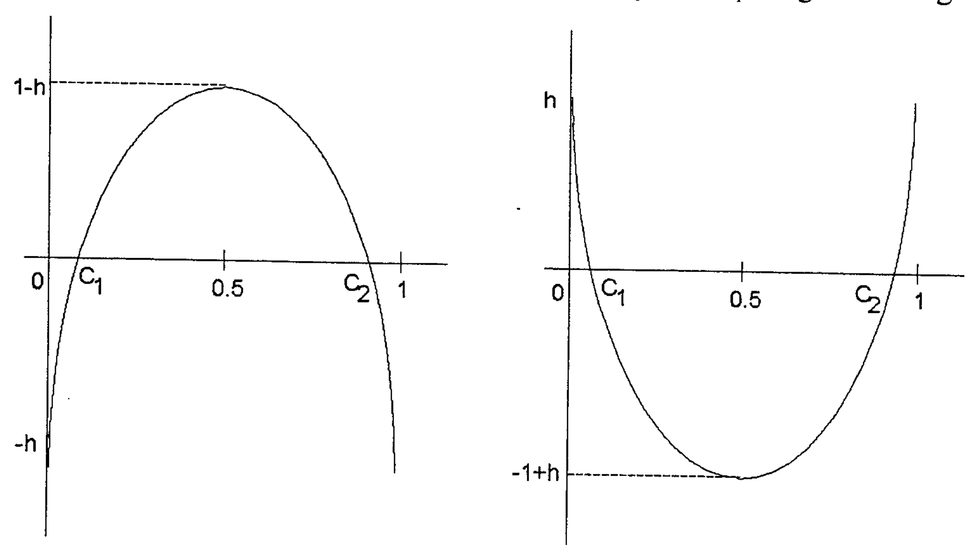
To represent these statements in the terms of the model, we introduce the following function:

$$U_a(x,h) = a(H(x) - h), \tag{12.1}$$

where $a = \pm 1$, $H = -x\log_2 x - (1-x)\log_2(1-x)$ and $h \in [0,1]$.

The value of function $U_a(x,h)$ will be called *potential of choice*; we will suppose that the agent accepts the task of choosing only under conditions that the potential of choice is not negative.

We can see that $H(x)$ is the entropy of choice. The value a=1 corresponds to agent I, and the value a=-1 to agent II. Examples of graph for U_1 and U_{-1} are given in Fig.12.1:



(a) $U_1 = H(x) - h$ (b) $U_{-1} = -H(x) + h$

Fig. 12.1. Graphs of the potentials of choice.
(a) Graph for agent I; the potential is not negative only at the interval $[c_1,c_2]$.
(b) Graph for agent II; the potential is not negative for intervals $[0,c_1]$ and $[c_2,1]$.

We see from Fig.12.1(a) that agent I accepts the task of choice only if his readiness belongs to the interval $[c_1,c_2]$, and from Fig.12.1(b) that agent II accepts the task of choice if his readiness belongs to the intervals $[0,c_1]$ and $[c_2,1]$. The value of $H(x)$ is an uncertainty measured in bits; we will consider h as a quantity of bits and give its interpretation. For agent I, h is the minimal uncertainty given which his cognitive system can make a choice. For agent II, h is the maximal uncertainty given which he still can make a choice. For agent I, the greater the value of h, the narrower the interval $[c_1,c_2]$ from which values of x can be taken. When h=0, the value of x can be any number from the interval [0,1]; when h=1, there is only one value of x, (½). For agent II, the greater the value of h, the wider the intervals $[0,c_1]$ and $[c_2,1]$, from which values of x can be taken. When h=0, x is either 0 or 1; when h=1, x can be any number from the interval [0,1]. Parameter h comprises the agent's individual characteristics defining the area of values for his readiness. The framework of mathematical modeling by itself does not permit us to construct practical methods for determining the values of h and a. This task must involve experts' evaluations.

In our previous report (Lefebvre, 1999b), we bring examples of reflexive control in which a controlling party took into consideration the limit values $P_{\max}$ and $P_{\min}$. The difference between agent I and agent II, however, was not then analyzed. We will investigate now what new opportunities may arise for reflexive control if we know the type of the agent (I or II). Let us return to the example given at the end of section 8. Country **A** evaluates positively only those outcomes, when **A**'s and **B**'s choices coincide, and x_1=x_2=0. **A**'s state of readiness in this case is given by (8.12). If agent **A** is not certain that **B** will make the demand for indictment $(y \neq 1)$, **A**'s readiness is (½). Let the criminal syndicate know that **A** is type II agent, i.e., a=-1 and his potential of choice is described by curve 12.1(b). Let also the syndicate be able to manipulate **A**'s image of **B**, i.e., to manipulate with y. What actions must undertake the syndicate to make **A**'s decision more difficult? It follows from a=-1 that the readiness (½) cannot be realized when h>0. The syndicate may not know the value of h. Nevertheless it must make A to doubt in B's demand $y \neq 1$. Then if it occurs that h>0, **A**'s cognitive system will not be able to generate readiness. If **A** remains certain that **B** will make the demand (y=1), then he would have freedom of choice, i.e., his cognitive system can generate readiness; x can be any number from $[0,c_1]$ or $[c_2,1]$.

13. THE GOLDEN SECTION

In accordance with the $< x_1, x_2 x_3 >$- model, the golden section value, $x = (\sqrt{5} - 1)/2 = 0.618...$, appears under the following conditions:

(1) The alternatives are strictly polarized.

(2) The agent does not have an operational criterion which would enable his

cognitive system to compute the utility-measures of the alternatives.
(3) The agent has no previous experience of choice in similar situations.
(4) The agent's choice is intentional.

The following equation corresponds to the $< x_1, x_2 x_3 >$ - model:

$$x = x_1 + (1 - x_1)(1 - x_2)x_3. \tag{13.1}$$

Since no operational criterion exists, pressures toward different poles are indistinguishable in their strength, so $x_1=(½)$, and since there is no previous experience, value of x_2 is determined by readiness x, so $x_2=x$. By substituting these values into (13.1) we obtain

$$x = \frac{1 + x_3}{2 + x_3}. \tag{13.2}$$

For the intentional choice, $x=x_3$, and (13.2) is transformed into

$$x^2 + x - 1 = 0, \tag{13.3}$$

whose positive root is $x = (\sqrt{5} - 1)/2 = 0.618...$ (see Lefebvre, 1997).

Consider an example. The UN announces a discussion on such an abstract problem as the prohibition of private ownership of land on the Moon. For country **A** this prohibition plays the role of the positive pole, but since no "measurable" factors in favor of this point of view exist, in accordance with the $< x_1, x_2 x_3 >$ - model, **A**'s favorable arguments constitutes 62%; that is, the expected pressure toward the positive pole is $x_2=0.62$ (Adams-Webber, 1997). During the UN discussion, four countries exert pressure on **A**, inclining it to vote against the prohibition, and one country tried to force country **A** to vote for the prohibition. So the actual pressure toward **A**'s positive pole is $x_1=0.2$. By substituting the values of x_1 and x_2 into (13.1) and assuming that $x_3=x$, we obtain the probability with which **A** will vote for the prohibition of private ownership of land on the Moon:

$$x = \frac{x_1}{x_1 + x_2 - x_1 x_2} \approx 0.29 .$$

This example illustrates that the value of x_2 equal to the golden section is chosen in those cases in which previous experience does not rest on measurable factors.

14. CONCLUSION

The analysis conducted in this research demonstrates the following:

(1) The model $<x_1,x_2,M>$ can be used to represent the interaction of two agents.
(2) The Boolean version of the $<x_1,x_2,M>$-model allows us to represent the agent's

generation of strategic decisions, i.e., ramified programs of his future behavior in IW.
(3) The Boolean-linear version of the $<x_1,x_2,M>$-model allows us to represent the agent's state at the moment of choice between the two poles.
(4) Both the Boolean version and Boolean-linear version of the model can be employed for conducting reflexive control in IW.
(5) For practical needs it seems necessary to create a special computerized program for modeling situations with the help of $<x_1,x_2,M>$.
(6) The next step in the theoretical research is to find the limits on application of the Boolean-linear version of the $<x_1,x_2,M>$-model and moving to the development of a nonlinear version of this model.

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